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INVENTORY POLICIES WITH RETAILER'S FLEXIBLE PAYMENT TIME AND CUSTOMER'S FIXED CREDIT TIME FOR MANUFACTURER-RETAILER SUPPLY CHAIN

Abstract. *An inventory model with price sensitive and time dependent demand to obtain joint inventory policies for a manufacturer retailer supply chain is developed in this paper. Products in the system are considered to be deteriorated at a constant rate. Manufacturer adopts a lot-for-lot policy for delivering retailer's demand and offers the retailer payment time dependent price for the product. Payment time dependent discount is offered to the retailer in case of advanced payment and payment time dependent higher product price is taken from retailer in case of delayed payment. Demand for retailer's side is considered to be price sensitive as well as time dependent quadratic in nature. Retailer gives a fixed trade credit to the customers. Optimal production quantity, payment time for retailer, individual profit for retailer and manufacturer and joint profit of the supply chain are discussed.*

Keywords*: Inventory model; Deterioration; Advance payment; Delayed payment; Selling price.*

JEL Classification: C5

1. Introduction

Credit period for the payment plays significant role in increasing total sales in present market scenario. Retailer can choose to pay to manufacturer in advance to enjoy the benefit of the reduced price to increase his profit or he can pay to manufacturer later after getting the replenishment where he get the product at a higher than the normal product price. This flexibility of payment gives strength to manufacturer retailer relationship. Further if the retailer has enough funds he may

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pay to manufacturer in advance and can take advantage to stock the product at a lower price and it can be useful to him to stay in a competitive market. On the other hand, if retailer do not have sufficient fund then by choosing delayed payment option still, he can stock the product at higher rate than regular price, then by selling them he can generate revenue and can complete payment to manufacturer. Retailer offers fixed credit period for payment to customers. This strategy is also benefit worthy to have strong bonding with customers and to increase sales of the product.

Ho et al. (2011) studied optimal inventory policy with price and credit linked demand under two level trade credit. Shah et al. (2014) carried out optimal pricing and ordering policies for deteriorating items with two level trade credits. They considered the demand of the product to be price sensitive and quadratic. They also allowed customer return for the product. Pervin et al. (2016), Wang et al. (2016) and Zhang et al. (2016) investigated optimum policies with various conditions for inventory models having trade credits. Teng et al. (2016) studied inventory lot size policies for deteriorating items with expiration dates and advance payments. Mahata et al. (2017) gave optimal replenishment and credit policy in a supply chain inventory model with two level trade credits. They considered time and credit-sensitive demand involving default risk. Wu et al. (2017) studied a supply chain model with trade credit and default risk. Li et al. (2017) gave pricing and lot sizing policies for perishable items with advance cash credit payments by a discounted cash-flow analysis. Diabat et al. (2017) investigated a lot sizing model with partial downstream delayed payment and partial upstream advance payment for deteriorating items. In their study they also considered partial back ordering for the products. Tiwari et al. (2018), Giri et al. (2018) and Shah et al. (2018) investigated inventory model with two level trade credits with different demand patterns. Ortodi at al. (2019) studied joint pricing and lot sizing for perishable item under two level trade credits with multiple demand class. Chang et al. (2019) studied manufacturer's pricing and lot sizing decisions under various payment terms by a discounted cash-flow analysis. Li et al. (2019) and Tsao et al. (2019) gave inventory model under advance cash credit payment scheme.

In this paper, we consider single manufacturer single retailer supply chain. Manufacturer adopts lot-for-lot policy to meet retailer's demand. Products in the system are considered to be deteriorated at a constant rate. Customer's demand at retailer's side is taken to be price sensitive as well as time dependent quadratic in nature. Manufacturer offers three choices to retailer for the payment. (i) Advance payment with discounted product price. (ii) Payment at the time of delivery of the product at normal product price. (iii) Delayed payment at a higher price than the normal product price. Further the retailer offers a fixed credit period for payment to customers. With consideration of all these parameters we study optimal policies for manufacturer retailer supply chain.

The paper is organized as: Section 1 is introduction. Assumptions and notations are stated in Section 2 which are used to develop the inventory model.

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Mathematical model for supply chain is developed in section 3. The results are validated in Section 4 through numeric hypothetical inventory parametric values and the managerial issues are discussed through sensitivity analysis, where one inventory parameter is varied by Section 5 concludes the study.

2. Assumptions and Notations:

For the development of mathematical model, we consider following assumptions and notations.

2.1 Assumptions

(1) Replenishment rate is infinite and there is no lead time.

(2) Manufacturer follows lot-for-lot production policy in response to retailer's ordering quantity.

(3) Customer demand for retailer side price sensitive and time dependent quadratic in nature. $R = R(p,t) = a\cdot\left(1+b\cdot t-c\cdot t^2-\alpha\cdot p\right)$ where $a>0, 0<b<1$ and $0<c<1$. $\alpha>0$ is scale parameter to the selling price such that $p<\dfrac{a}{\alpha}$.

(4) Manufacturer offers flexible payment choice to the retailer.

(i) Advance payment before replenishment of product. Here retailer gets the product at discounted price w_1. where $w_1=(1-\delta\cdot M)cr$ with $0<\delta<1$.

(ii) Payment at the time of replenishment or delayed payment after the replenishment. In case of payment at the time of replenishment retailer gets the product at normal price and in case of delayed payment after replenishment he gets the product at a higher than normal product price. Both can be represented by $w_2=(1+\delta\cdot M)cr$ with $\delta>0$.

(5) Product is considered to be deteriorated at a constant rate θ.there is no replacement or repair for deteriorated items in the inventory system.

(6) Retailer need to borrow money from financial corporation at interest rate ic whenever he needs to pay in advance and earns interest at the interest rate ie by depositing selling revenue to bank or financial corporation.

(7) The manufacturer incurs an opportunity loss at an interest rate iv .

(8) Shortages are not allowed.

2.2 Notations:

A_m	Manufacturer's set up cost in \$ / order
A_r	Retailer's ordering cost in \$ / order
$I_m(t)$	Inventory level at time t for manufacturer
$I_r(t)$	Inventory level at time t for retailer
M	Payment time for retailer to settle account with manufacturer
N	Credit period offered by retailer to customers
P	Production rate for manufacturer

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Q	Retailer's order quantity
R	Customer's demand rate for retailer side
T	Cycle length
cm	Manufacturer's cost price in \$ / unit
cr	Retailer's cost price in \$ / unit for payment at replenishment time
w_1	Retailer's cost price in \$ / unit for advance payment
w_2	Retailer's cost price in \$ / unit for delayed payment
hm	Manufacturer's holding cost in \$ per unit per unit time
hr	Retailer's holding cost in \$ per unit per unit time
ie	Interest rate at which interest is earned
ic	Interest rate at which interest is charged $(ic > ie)$
iv	Interest rate lost by manufacturer in case of delayed payment
p	Customer's selling price in \$ / unit (decision variable)
δ	Factor associated with retailer's payment time
θ	Rate of deterioration

3 Mathematical Model:

We consider separate calculations for the manufacturer and retailer's individual profit and then proceed to evaluate average profit for the supply chain for given cycle.

3.1 Manufacturer's perspective:

Production rate of manufacturer is P so the time taken to meet the retailer's requirement is $T_1 = \frac{Q}{P}$ Manufacturer's inventory level at time t is governed by the differential equation,

$$\frac{d}{dt} I_m(t) = P \; ; \; 0 \le t \le T_1 \tag{1}$$

Solving this using the initial condition $I_m(0) = 0$ we get, $I_m(t) = P \cdot t$ (2)

Holding cost is given by, $HC_m = hm\int_0^{T_1} I_m(t)\,dt$ and Set up cost $OC_m = A_m$

According to retailer's payment time we have following cases for calculation of manufacturer's total profit.

Case 1: Advance Payment

In this case manufacturer offers product to retailer at discounted price w_1. Manufacturer then keeps this advance payment in a financial corporation or bank to earn interest at the rate ie and to recover the loss in profit due to lower selling price. Interest earned by manufacturer is $IE_{m1} = ie \cdot w_1 \cdot Q \cdot M$ and sales revenue is $SR_{m1} = (w_1 - cm) \cdot Q$. So the profit is given by,

$$\pi_{m1} = \frac{1}{T}\left(SR_{m1} + IE_{m1} - OC_m - HC_m\right) = \frac{1}{T}\left((w_1 - cm)Q + ie \cdot w_1 QM - A_m - hm\int_0^{T_1} I_m(t)\,dt\right) \tag{3}$$

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Case 2: Delayed Payment

In this case retailer pays later after some time of the replenishment. Manufacturer charges higher product price w_2 and gains more from retailer. But during this time the manufacturer may have the rolling money which is not achieved in reality. Thus the corresponding opportunity loss is $iv \cdot w_2 \cdot Q \cdot M$ and sales revenue is $SR_{m2} = (w_2 - cm) \cdot Q$. So, the profit is given by,

$$\pi_{m2} = \frac{1}{T}\left(SR_{m2} - OC_m - HC_m - OL_m\right) = \frac{1}{T}\left(\left(w_2 - cm\right)Q - A_m - hm\int_0^{T_1} I_m(t)\,dt - iv \cdot w_2 QM\right) \quad (4)$$

3.2 Retailer's perspective

Retailer's inventory level decreases due to market demand and deterioration. Hence inventory at any time is governed by the differential equation,

$$\frac{d}{dt} I_r(t) = -R(p,t) - \theta \cdot I_r(t)\ ;\ 0 \le t \le T \quad (5)$$

Using boundary condition $I_r(T) = 0$ and solving we get,

$$I_r(t) = a \cdot \left[\begin{array}{l} \dfrac{\left(1 + b \cdot T - c \cdot T^2 - \alpha \cdot p\right)e^{\theta \cdot (T-t)}}{\theta} - \dfrac{\left(b - 2c \cdot T\right)e^{\theta \cdot (T-t)}}{\theta^2} - \\ \dfrac{2c \cdot e^{\theta \cdot (T-t)}}{\theta^3} - \dfrac{\left(1 + b \cdot t - c \cdot t^2 - \alpha \cdot p\right)}{\theta} + \dfrac{\left(b - 2c \cdot t\right)}{\theta^2} + \dfrac{2c}{\theta^3} \end{array}\right] \quad (6)$$

Further, using $I_r(0) = Q$ we get the ordering quantity for retailer as,

$$Q = a \cdot \left[\frac{\left(1 + b \cdot T - c \cdot T^2 - \alpha \cdot p\right)e^{\theta \cdot T}}{\theta} - \frac{\left(b - 2c \cdot T\right)e^{\theta \cdot T}}{\theta^2} - \frac{2c \cdot e^{\theta \cdot T}}{\theta^3} - \frac{\left(1 - \alpha \cdot p\right)}{\theta} + \frac{b}{\theta^2} + \frac{2c}{\theta^3}\right] \quad (7)$$

Ordering cost is $OC_r = A_r$ and the holding cost is, $HC_r = hr \cdot \int_0^T I_r(t)\,dt$.

Retailer is allowed to make choice of payment time M to the manufacturer. We have different cases on the basis of retailer's payment time M, credit period offered to customers N and cycle time T.

Case 1: Advance Payment

In this case the retailer pays to the manufacturer before the replenishment. Retailer borrows money from market and pays to manufacturer in advance, so till the time $T + M$ he is charged interest on borrowed money. Therefore, $IC_1 = w_1 \cdot ic \cdot (T + M) \cdot Q$

Selling revenue is kept in bank to generate revenue from interest, which earns interest $IE_1 = ie \cdot p \cdot \int_0^T R(p,t) \cdot t\,dt$. Due to credit period offered to the customers the

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retailer incurs opportunity loss, $OL_1 = iv \cdot p \cdot \int_0^N R(p,t) \cdot t\,dt$. Total revenue generated is $SR_1 = (p - w_1) \cdot \int_0^T R(p,t) \cdot t\,dt$. So the total profit is,

$$\pi_{r1} = \frac{1}{T}\left(SR_1 - OC_r - HC_r + IE_1 - IC_1 - OL_1\right)$$
$$= \frac{1}{T}\begin{pmatrix} (p - w_1) \cdot \int_0^T R(p,t) \cdot t\,dt - A_r + ie \cdot p \cdot \int_0^T R(p,t) \cdot t\,dt - \\ w_1 \cdot ic \cdot (T + M) \cdot Q - iv \cdot p \cdot \int_0^N R(p,t) \cdot t\,dt - hr \cdot \int_0^T I_r(t)\,dt \end{pmatrix} \tag{8}$$

Case 2: Delayed Payment

In this case the retailer pays to manufacturer after some time of receiving replenishment. Due to delayed payment he receives product at higher price w_2. Thus revenue generated is given by,

$SR_2 = (p - w_2) \cdot \int_0^T R(p,t) \cdot t\,dt$.

We have following sub cases depending upon values of M, N and $T + N$.

(i) Sub case 2A : $N < M$ and (ii) Sub case 2B: $M < N$

Sub case 2A : $N < M$

In this case credit period offered to customers is less than the retailer’s payment time. This lead to following sub cases.

Sub case 2A1 : $N < M < T + N$

Interest earned by retailer in this sub case is $IE_{2a1} = ie \cdot p \cdot \int_0^{M-N} R(p,t) \cdot t\,dt$.Due to payment at time M the retailer is charged interest as $IC_{2a1} = ic \cdot w_2 \cdot \int_M^{T+N} I_r(t)\,dt$.

Hence total profit of the retailer for this sub case is given by,

$$\pi_{2a1} = \frac{1}{T}\left(SR_2 - OC_r - HC_r + IE_{2a1} - IC_{2a1}\right)$$
$$= \frac{1}{T}\left((p - w_2) \cdot \int_0^T R(p,t) \cdot t\,dt - A_r - hr \cdot \int_0^T I_r(t)\,dt + ie \cdot p \cdot \int_0^{M-N} R(p,t) \cdot t\,dt - ic \cdot w_2 \cdot \int_M^{T+N} I_r(t)\,dt\right) \tag{9}$$

Sub case 2A2 : $N < T + N < M$

In this sub case the retailer do not need to pay interest as the payment from retailer’s side is done after receiving complete payment for the cycle so $IC_{2a2} = 0$. Retailer starts getting revenue from customers at time N and till the time $T + N$ he

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earns interest on average sales volume. For the time period $T+N$ to M, he earns interest on full sales revenue. $IE_{2a2} = ie \cdot p \cdot \left[\int_0^T R(p,t) \cdot t\,dt + Q \cdot (M-T-N) \right]$.Profit is

$$\begin{aligned} \pi_{2a2} &= \frac{1}{T}\left(SR_2 - OC_r - HC_r + IE_{2a2} - IC_{2a2}\right) \\ &= \frac{1}{T}\left((p-w_2) \cdot \int_0^T R(p,t) \cdot t\,dt - A_r - hr \cdot \int_0^T I_r(t)\,dt + ie \cdot p \cdot \left(\int_0^T R(p,t) \cdot t\,dt + Q \cdot (M-T-N) \right) \right) \end{aligned} \tag{10}$$

Sub case 2B: $M < N$

In this sub case we have $M < N < T+N$. Payment from retailer's side will be completed before receiving revenue from customers and hence $IE_{2b} = 0$. During $[M,N]$ retailer is charged interest on full quantity and for $[N, T+N]$ he is charged interest on average inventory. $IC_{2b} = ic \cdot w_2 \cdot \left[Q \cdot (N-M) + \int_o^T I_r(t)\,dt \right]$ so profit is,

$$\begin{aligned} \pi_{2b} &= \frac{1}{T}\left(SR_2 - OC_r - HC_r + IE_{2b} - IC_{2b}\right) \\ &= \frac{1}{T}\left((p-w_2) \cdot \int_0^T R(p,t) \cdot t\,dt - A_r - hr \cdot \int_0^T I_r(t)\,dt - ic \cdot w_2 \cdot \left(Q \cdot (N-M) + \int_o^T I_r(t)\,dt \right) \right) \end{aligned} \tag{11}$$

3.3 Assimilation to supply chain

So far we discussed total profit for manufacturer and retailer individually. Now we proceed to assimilate them to obtain profit for the supply chain as follow:

$$\pi = \begin{cases} \pi_1; & \text{For advance payment} \\ \pi_2; & \text{when } N < M < T+N \\ \pi_3; & \text{when } N < T+N < M \\ \pi_4; & \text{when } M < N < T+N \end{cases} \tag{12}$$

where,

$$\begin{aligned} \pi_1 &= \pi_{m1} + \pi_{r1} \\ &= \frac{1}{T}\left[\begin{pmatrix} (w_1 - cm)Q + ie \cdot w_1 QM \\ -A_m - hm \int_0^{T_1} I_m(t)\,dt \end{pmatrix} + \begin{pmatrix} (p-w_1) \cdot \int_0^T R(p,t) \cdot t\,dt - A_r + ie \cdot p \cdot \int_0^T R(p,t) \cdot t\,dt - \\ w_1 \cdot ic \cdot (T+M) \cdot Q - iv \cdot p \cdot \int_0^N R(p,t) \cdot t\,dt - hr \cdot \int_0^T I_r(t)\,dt \end{pmatrix} \right] \end{aligned} \tag{13}$$

$$\begin{aligned} \pi_2 &= \pi_{m2} + \pi_{2a1} \\ &= \frac{1}{T}\left[\begin{pmatrix} (w_2 - cm)Q - iv \cdot w_2 QM \\ -A_m - hm \int_0^{T_1} I_m(t)\,dt \end{pmatrix} + \begin{pmatrix} (p-w_2) \cdot \int_0^T R(p,t) \cdot t\,dt - A_r - hr \cdot \int_0^T I_r(t)\,dt \\ +ie \cdot p \cdot \int_0^{M-N} R(p,t) \cdot t\,dt - ic \cdot w_2 \cdot \int_M^{T+N} I_r(t)\,dt \end{pmatrix} \right] \end{aligned} \tag{14}$$

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$$\pi_3 = \pi_{m2} + \pi_{2a2}$$

$$= \frac{1}{T}\left[\begin{pmatrix} (w_2 - cm)Q - iv \cdot w_2 QM \\ -A_m - hm\int_0^{T_1} I_m(t)\,dt \end{pmatrix} + \begin{pmatrix} (p - w_2)\cdot\int_0^T R(p,t)\cdot t\,dt - A_r - hr\cdot\int_0^T I_r(t)\,dt \\ +ie\cdot p\cdot\left(\int_0^T R(p,t)\cdot t\,dt + Q\cdot(M - T - N)\right) \end{pmatrix}\right] \tag{15}$$

$$\pi_4 = \pi_{m2} + \pi_{2b}$$

$$= \frac{1}{T}\left[\begin{pmatrix} (w_2 - cm)Q - iv \cdot w_2 QM \\ -A_m - hm\int_0^{T_1} I_m(t)\,dt \end{pmatrix} + \begin{pmatrix} (p - w_2)\cdot\int_0^T R(p,t)\cdot t\,dt - hr\cdot\int_0^T I_r(t)\,dt \\ -ic\cdot w_2\cdot\left(Q\cdot(N - M) + \int_o^T I_r(t)\,dt\right) - A_r \end{pmatrix}\right] \tag{16}$$

4. Numerical Example:

In this section we illustrate with numerical examples in order to validate the inventory model. The objective is to maximize total profit of the supply chain. First, we calculate optimum values of decision variables M and p. To do so according to applicable case we work out partial derivatives of respective profit functions with respect to M and p. By setting them zero we get the required values. This is shown in following procedure.

Step 1: Allocate values to all inventory parameters other than decision variables.

Step 2: Work out $\frac{\partial \pi_i}{\partial M} = 0$ *and* $\frac{\partial \pi_i}{\partial p} = 0$; for appropriate i to get optimum values of decision variables M and p respectively.

Step 3: In order to check concavity of profit function work out the Hessian matrix,

$$H = \begin{bmatrix} \frac{\partial^2 \pi_i}{\partial M^2} & \frac{\partial^2 \pi_i}{\partial M\,\partial p} \\ \frac{\partial^2 \pi_i}{\partial p \partial M} & \frac{\partial^2 \pi_i}{\partial p^2} \end{bmatrix} \text{ Calculate } D_1 = \frac{\partial^2 \pi_i}{\partial M^2} \text{ and } D_2 = \begin{vmatrix} \frac{\partial^2 \pi_i}{\partial M^2} & \frac{\partial^2 \pi_i}{\partial M\,\partial p} \\ \frac{\partial^2 \pi_i}{\partial p \partial M} & \frac{\partial^2 \pi_i}{\partial p^2} \end{vmatrix}$$

In order to get concavity of the profit function, H should be negative definite.

H is negative definite if $D_1 < 0$ *and* $D_2 > 0$. Or the concavity can be checked through graph also.

Step 4: Substitute values of decision variables obtained above in equation (13), (14), (15) or (16)as per applicable case to get optimum value of total profit of supply chain. By substituting the values in equation (7) we get optimum production quantity.

We consider following examples to validate the mathematical formulation.

Example 1(Advance Payment): Consider $A_m = \$\,500$ *and* $A_r = \$\,350$ per order, $a = 100$, $b = 0.4$, $c = 0.3$ $\alpha = 0.002$, $cm = \$150$ /unit, $cr = \$250$ /unit, $hm = \$1.5$

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/unit/month, $hr = \$1.8$ /unit/month, $ie = 0.15$, $ic = 0.28$, $iv = 0.03$, $P = 600$ units/unit time, $T = 0.5$, $N = 0.2$, $\delta = 0.4$ and $\theta = 0.1$.

Following the procedure mentioned we get optimal values $M = 0.71$ before the replenishment, selling price $p = \$\ 349.28 / unit$. For concavity of the profit function we work out D_1 *and* D_2. $D_1 = -369.77 < 0$ and $D_2 = 870.59 > 0$ which ensures concavity. Further the concavity can be seen from graph 1 in figure 1.

Example 2 (Delayed Payment - $N < M < T + N$): consider $T = 2.5, \alpha = 0.003, N = 0.8$ and $\delta = 0.12$. All other parameters are same as given in example 1. With these values we obtain optimum payment time $M = 0.82$ and $p = 268.31$. By following step 3 we get $D_1 = -1847.6 < 0$ and $D_2 = 1222.63 > 0$ that ensures concavity and it is also shown in graph 2 in figure 1.

Example 3 (Delayed Payment - $N < T + N < M$): consider $T = 0.15$, $N = 0.03, c = 0.2$ and $\delta = 0.2$. All other parameters are same as given in example 1. With these values we obtain optimum payment time $M = 14.99$ and $p = 349.91$. By following step 3 we get $D_1 = -99.36 < 0$ and $D_2 = 104.8 > 0$ that ensures concavity and it is also shown in graph 1 in figure 2.

Example 4 (Delayed Payment - $M < N < T + N$): We consider $T = 3$, $N = 0.8, c = 0.1$ and $\delta = 0.45$. All other parameters are same as given in example 1. With these values we obtain optimum payment time $M = 0.23$ and $p = 481.24$. By following step 3 we get $D_1 = -4.66 < 0$ and $D_2 = 1057.73 > 0$ that ensure concavity and the concavity can also be seen in the graph 2 in figure 2.

Individual profits for retailer and manufacturer, total profit for supply chain and optimum values of payment time for retailer and selling price are stated in table 1.

Table 1. Optimum values for all cases

Case	M	P	Manufacturer's Profit π_m	Retailer's Profit π_r	Total Profit π
Advance Payment	0.71	349.28	604.36	3364.66	3969.02
Delayed Payment $N < M < T + N$	0.82	268.30	490.40	728.88	1219.28
Delayed Payment $N < T + N < M$	14.99	349.91	9915.49	2027.01	11942.5
Delayed Payment $M < N < T + N$	0.23	481.24	4837.44	1241.29	6078.73

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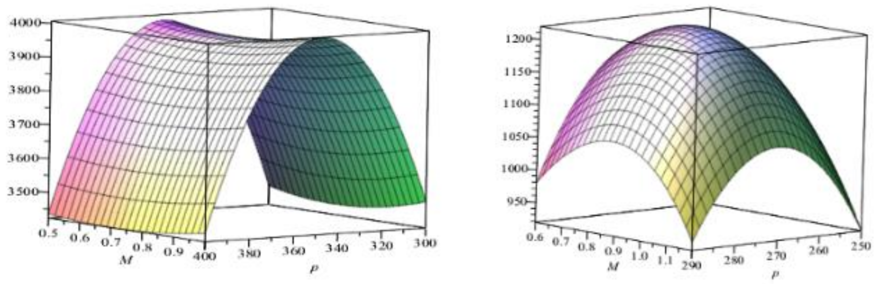
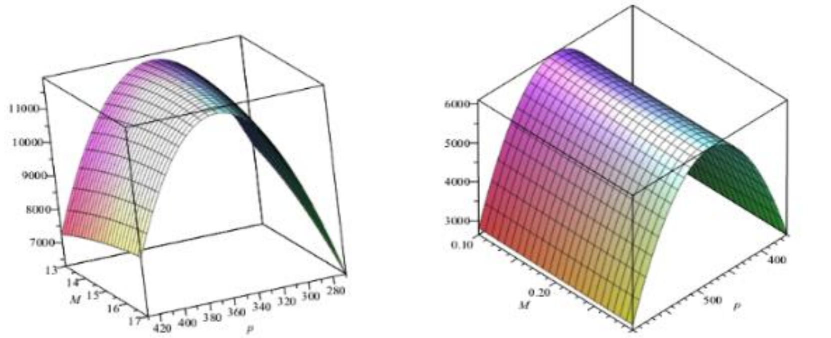


1 Concavity of Profit function (Advance Payment) 2 Concavity of Profit function (N < M < T+N)

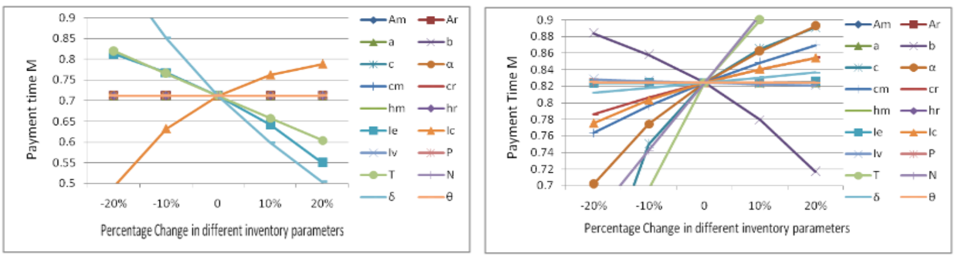
Figure 1. Concavity of Profit function with respect to decision variables



1 Concavity of Profit function (N < T+N < M) 2 Concavity of Profit function (M < N < T+N)

Figure 2. Concavity of Profit function with respect to decision variables

Next, we proceed to determine the sensitivity of Payment time, Selling price, Manufacturer's profit, Retailer's profit, Supply chain total profit and Order quantity with respect to change in other inventory parameters by $-20\%, -10\%, 10\%$ *and* 20% .



Sensitivity of Payment time M (Advance Payment) Sensitivity of Payment time M (N < M < T + N)

Figure 3. Sensitivity of Payment time M with respect to change in other parameters

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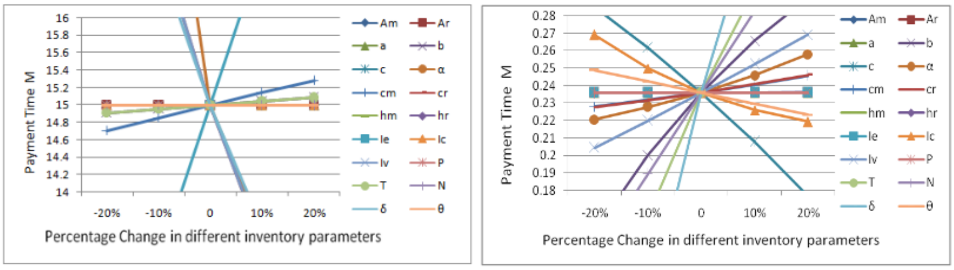
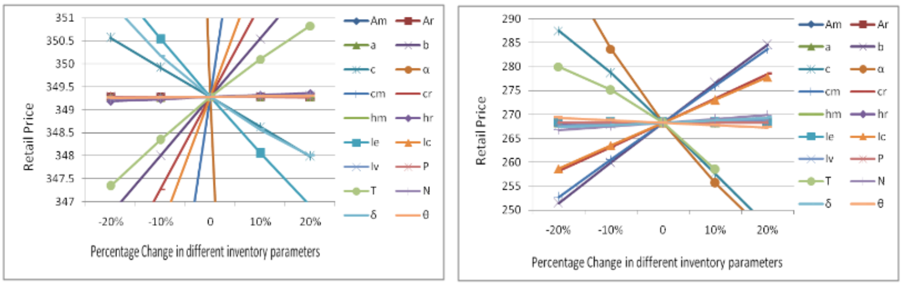


Sensitivity of Payment time M (N < T + N < M) Sensitivity of Payment time M (M < N < T + N)

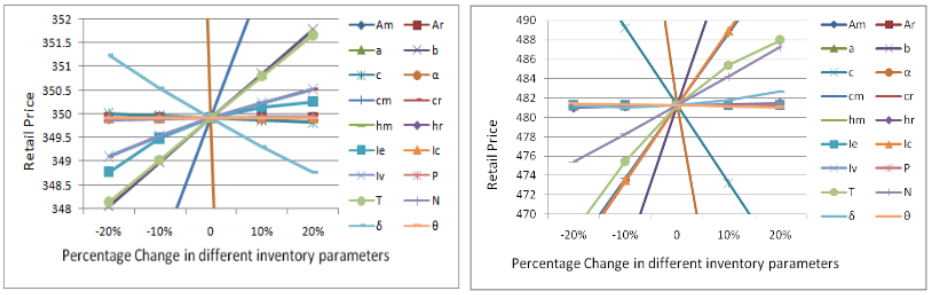
Figure 4. Sensitivity of Payment time M with respect to change in other parameters

Figure 3 and Figure 4 shows that, Payment time is very responsive to variables $b, c, \alpha, cm, cr, ie, ic, T, N$ and δ. There is negligible effect of change in the parameters A_m, A_r, hm, hr and θ on the payment time. With increase in values of cm, cr, b, T and N payment time for the retailer increases while increase in the rate of deterioration θ cause decrease in payment time. Other parameters have negligible effect on cycle time.



Sensitivity of Retail Price p (Advance Payment) Sensitivity of Retail Price p (N < M < T + N)

Figure 5. Sensitivity of Retail Price p with respect to change in other parameters



Sensitivity of Retail Price p (N < T + N < M) Sensitivity of Retail Price p (M < N < T + N)

Figure 6. Sensitivity of Retail Price p with respect to change in other parameters

Figure 5 and Figure 6 represents, Selling price is highly responsive to $b, c, \alpha, T, N < cm, cr, \delta$ and θ. Increase in ordering cost and holding cost for retailer

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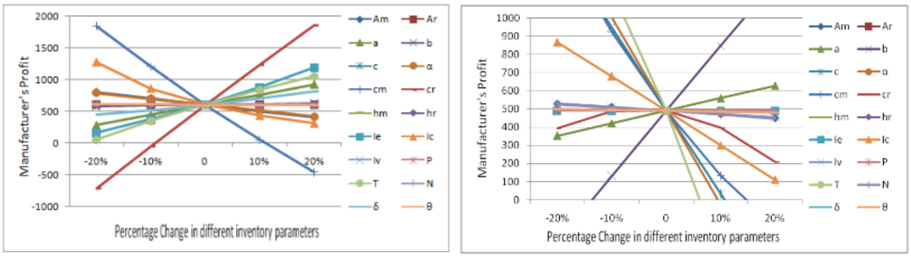
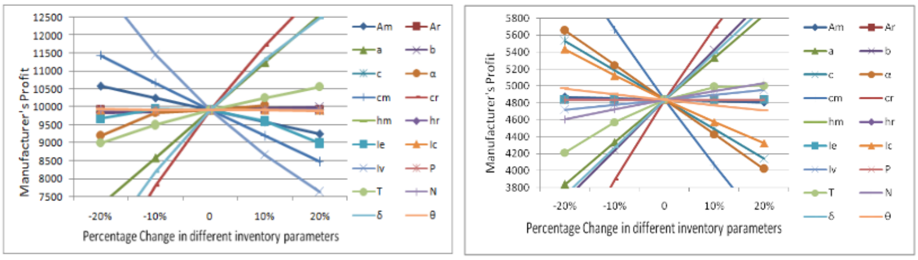
results into hike in selling price. If we look at demand components then increase in b results in increase of selling price while increase in c and α cause decrease in the selling price. Increase in costs cm, cr, hr, cycle time T, credit period N and deterioration rate θ also inspire a hike in the retail price.



Sensitivity of Manufacturer's Profit (Advance Payment) Sensitivity of Manufacturer's Profit (N < M < T + N)

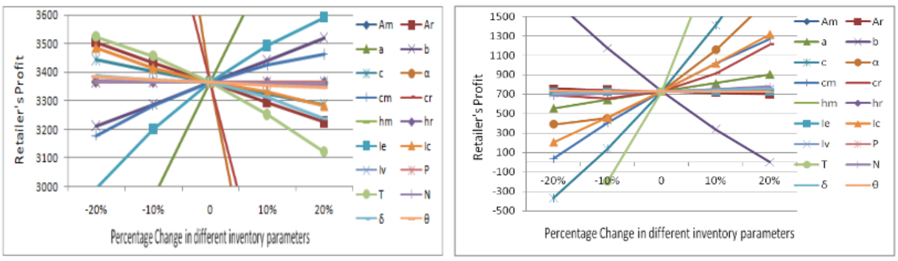
Figure 7. Sensitivity of Manufacturer's Profit with respect to change in other parameters



Sensitivity of Manufacturer's Profit (N < T + N < M) Sensitivity of Manufacturer's Profit (M < N < T + N)

Figure 8. Sensitivity of Manufacturer's Profit with respect to change in other parameters

Figure 7 and Figure 8 indicates, with increase in holding cost hm and the cost price cm for the manufacturer the manufacturer's profit decrease. On the other side cost price for the retailer is selling price for the manufacturer hence the increase in the cost price cr of retailer encourages the profit hike for manufacturer. Increase in T and δ also increase the profit. There is negligible effect of change in the parameters hr, A_r and θ on manufacturer's profit. The demand components a and b cause rise in the profit and c and α force a drop in the profit of manufacturer.



Sensitivity of Retailer's Profit (Advance Payment) Sensitivity of Retailer's Profit (N < M < T + N)

Figure 9. Sensitivity of Retailer's Profit with respect to change in other parameters

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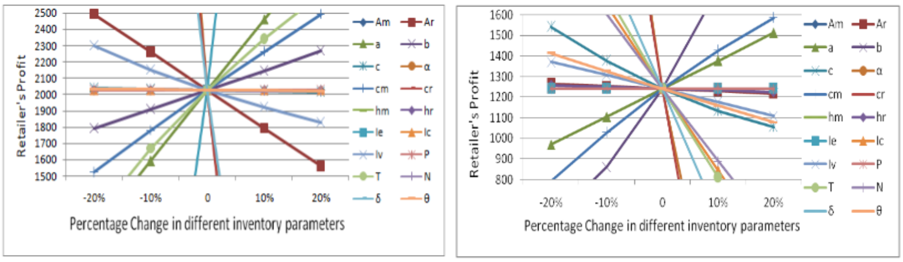
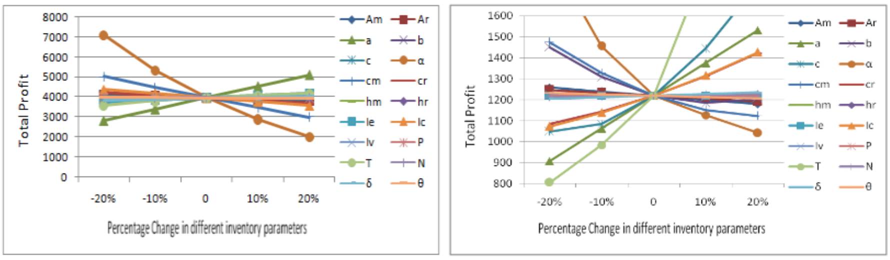


Sensitivity of Retailer's Profit (N < T + N < M)   Sensitivity of Retailer's Profit (M < N < T + N)

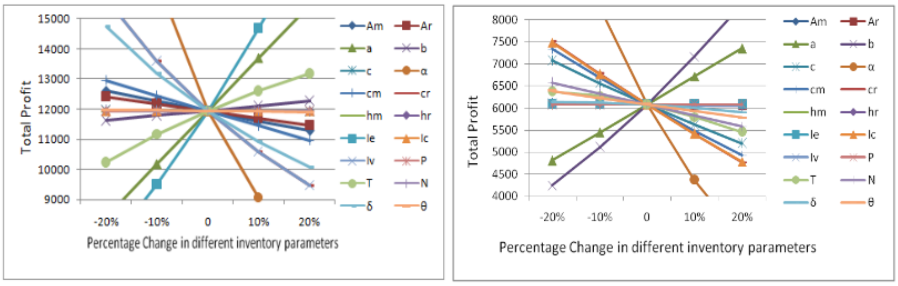
Figure 10. Sensitivity of Retailer's Profit with respect to change in other parameters

Figure 9 and Figure 10 express that, Retailer's profit remains almost un changes with the change in parameters like A_m, hm and P. It is very responsive to $A_r, a, b, c, \alpha, cm, cr, T, N$ and θ. Increase in cost price of retailer cr, holding price hr, c, α and the rate of deterioration θ result into decrease in retailer's profit. We may note here as the rate of deterioration increase naturally it force the decrease in the profit. Increase in demand components a and b encourage increase in retailer's profit.



Sensitivity of Supply Chain Profit (Advance Payment)   Sensitivity of Supply Chain Profit (N < M < T + N)

Figure 11. Sensitivity of Supply Chain Profit with respect to change in other parameters



Sensitivity of Supply Chain Profit (N < T + N < M)   Sensitivity of Supply Chain Profit (M < N < T + N)

Figure 12. Sensitivity of Supply Chain Profit with respect to change in other parameters

Figure 11 and Figure 12 represent, Total profit is very sensitive to the parameters $A_m, A_r, a, b, c, \alpha, cm, cr, T, N$ and θ. As the costs associated to inventory increase the

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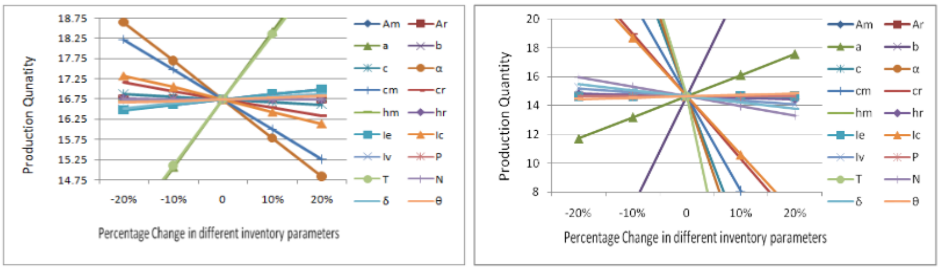
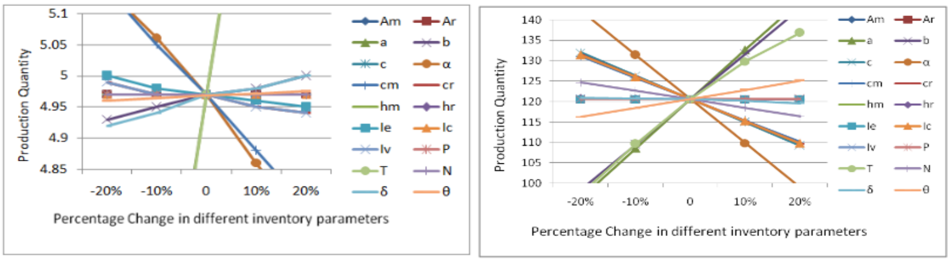
total profit of supply chain decrease hence increase in A_m, A_r, cm, cr, hm and hr causes a decrease in the total profit. Increase in c, α, θ and N results in decrease of the total profit. Increase a, b, T lead to increase in profit of the supply chain.



Sensitivity of Production Quantity (Advance Payment) Sensitivity of Production Quantity (N < M < T + N)

Figure 13. Sensitivity of Production Quantity with respect to change in other parameters



Sensitivity of Production Quantity (N < T + N < M) Sensitivity of Production Quantity (M < N < T + N)

Figure 14. Sensitivity of Production Quantity with respect to change in other parameters

Figure 13 and Figure 14 show that with increase in ordering cost the production quantity slightly increases while increase in holding cost and cost price of retailer and manufacturer results into decrease in the production quantity. Increase in θ, a and b result to increase in the production quantity while increase in the demand components c and α cause the decrease in the production quantity.

5. Conclusion:

We have studied single manufacturer single retailer supply chain for deteriorating items. We consider the flexibility for the retailer to pay to the manufacturer. Retailer may pay to the manufacturer in advance to enjoy the benefit of getting the product at a discounted rate or he may choose the delayed option for payment after the replenishment but here he has to pay higher price for the product than the normal price. Retailer also passes a fixed credit period to his customers. Thus considering payment time based cost price for retailer we study individual profits for retailer and manufacturer as well as the total profit for the supply chain. We consider demand of the product to be price sensitive and time dependent quadratic in nature. Numerical examples have been given to validate the model for each case. Sensitivity analysis has been carried out to check the effect of different parameters on decision variables.

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